TEKNIK DIGITAL (A) (TI 2104)

Materi Kuliah ke-5

BOOLEAN ALGEBRA AND LOGIC SIMPLICATION

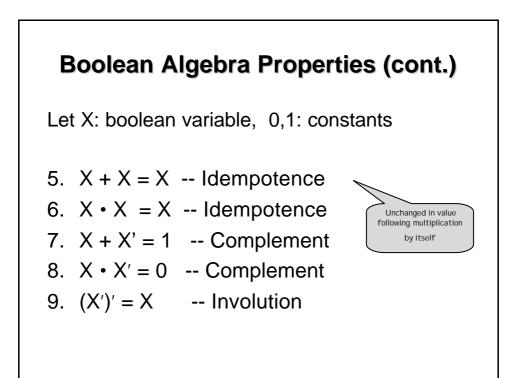
Boolean Algebra

- VERY nice machinery used to manipulate (simplify) Boolean functions
- George Boole (1815-1864): "An investigation of the laws of thought"
- Terminology:
 - Literal: A variable or its complement
 - Product term: literals connected by •
 - Sum term: literals connected by +

Boolean Algebra Properties

Let X: boolean variable, 0,1: constants

X + 0 = X -- Zero Axiom
 X • 1 = X -- Unit Axiom
 X + 1 = 1 -- Unit Property
 X • 0 = 0 -- Zero Property

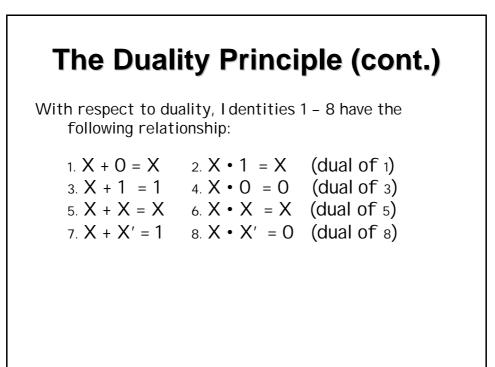


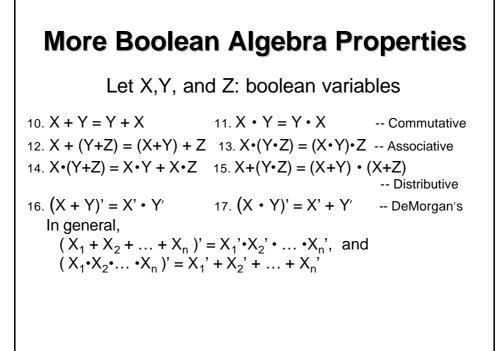
The Duality Principle

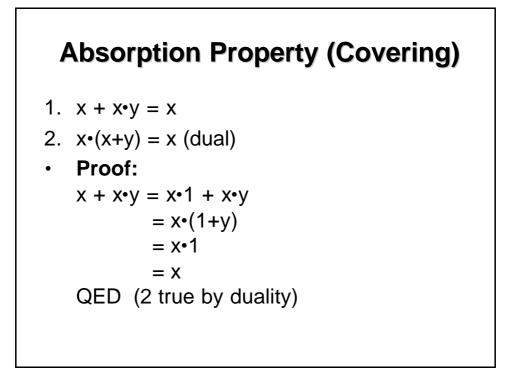
- The dual of an expression is obtained by exchanging (• and +), and (1 and 0) in it, provided that the precedence of operations is not changed.
- Cannot exchange x with x'

• Example:

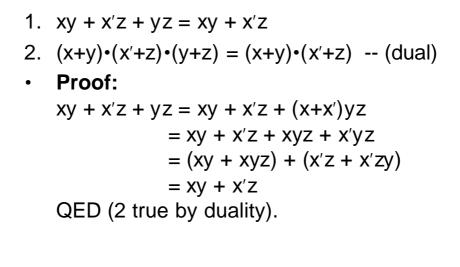
- Find H(x,y,z), the dual of F(x,y,z) = x'yz' + x'y'z
- H = (x'+y+z') (x'+y'+z)
- Dual does not always equal the original expression
- If a Boolean equation/equality is valid, its dual is also valid

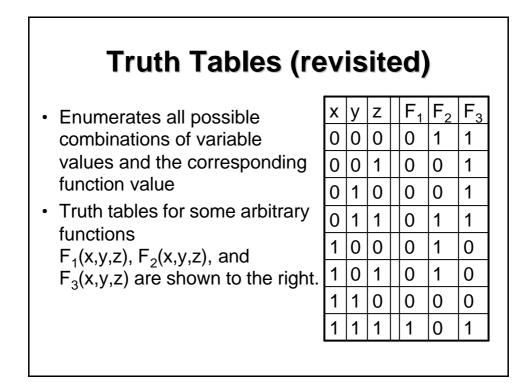


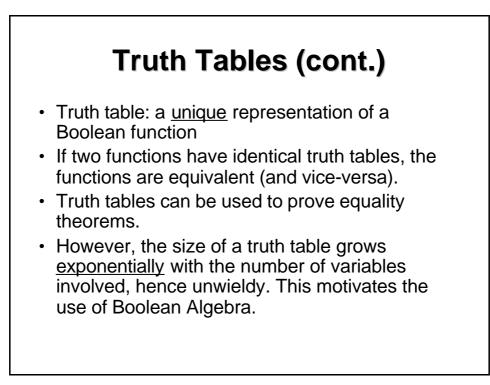


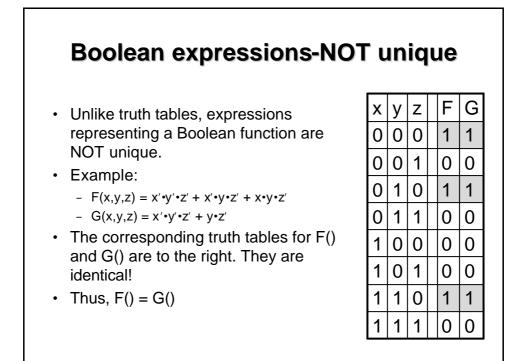


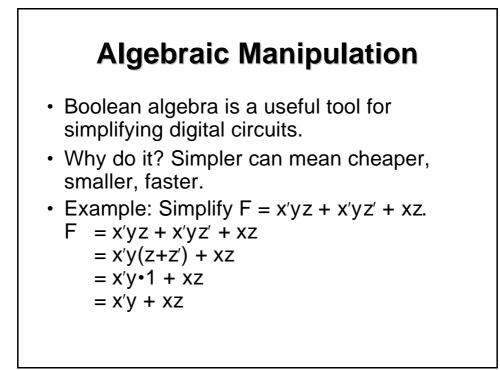
Consensus Theorem

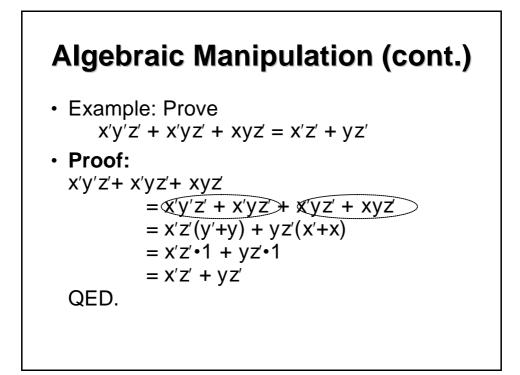












Complement of a Function

- The complement of a function is derived by interchanging (• and +), and (1 and 0), and complementing each variable.
- Otherwise, interchange 1s to 0s in the truth table column showing F.
- The *complement* of a function IS NOT THE SAME as the *dual* of a function.



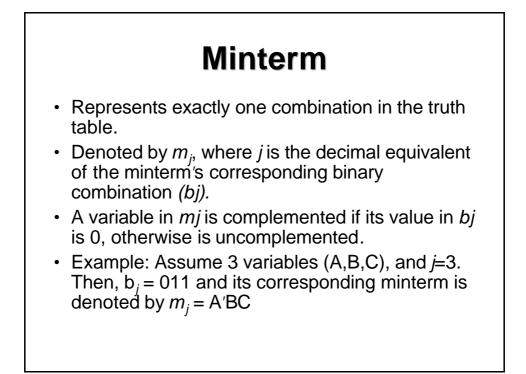
- Find G(x,y,z), the complement of F(x,y,z) = xy'z' + x'yz
- G = F' = (xy'z' + x'yz)'= $(xy'z')' \cdot (x'yz)'$ DeMorgan = $(x'+y+z) \cdot (x+y'+z')$ DeMorgan again
- Note: The complement of a function can also be derived by finding the function's *dual*, and then complementing all of the literals

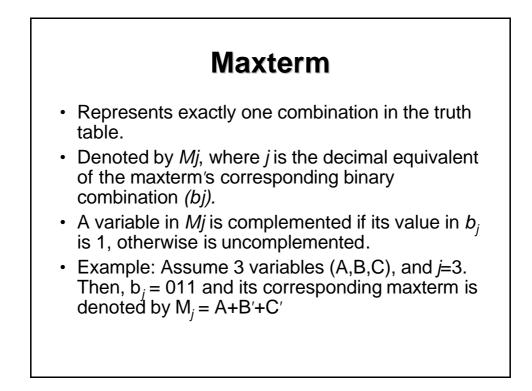
Canonical and Standard Forms

- We need to consider formal techniques for the simplification of Boolean functions.
 - Minterms and Maxterms
 - Sum-of-Minterms and Product-of-Maxterms
 - Product and Sum terms
 - Sum-of-Products (SOP) and Product-of-Sums (POS)

Definitions

- Literal: A variable or its complement
- Product term: literals connected by •
- Sum term: literals connected by +
- *Minterm:* a product term in which all the variables appear exactly once, either complemented or uncomplemented
- Maxterm: a sum term in which all the variables appear exactly once, either complemented or uncomplemented

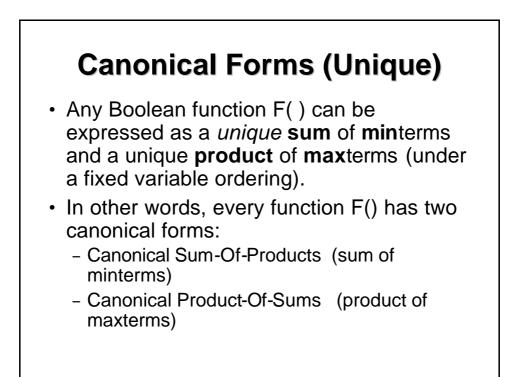




Truth Table notation for Minterms and Maxterms

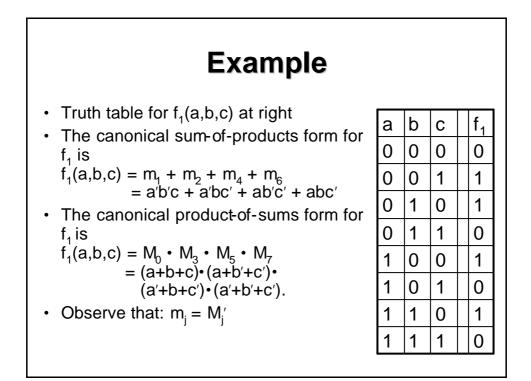
- Minterms and Maxterms are easy to denote using a truth table.
- Example: Assume 3 variables x,y,z (order is fixed)

х	у	z	Minterm	Maxterm
0	0	0	$x'y'z' = m_0$	$x+y+z = M_0$
0	0	1	$x'y'z = m_1$	$x+y+z' = M_1$
0	1	0	$x'yz' = m_2$	$x+y'+z = M_2$
0	1	1	x'yz = m ₃	$x+y'+z'=M_3$
1	0	0	$xy'z' = m_4$	$x'+y+z = M_4$
1	0	1	$xy'z = m_5$	$x'+y+z' = M_5$
1	1	0	$xyz' = m_6$	$x'+y'+z = M_6$
1	1	1	xyz = m ₇	$x'+y'+z' = M_7$



Canonical Forms (cont.)

- Canonical Sum-Of-Products: The minterms included are those m_j such that F() = 1 in row j of the truth table for F().
- Canonical Product-Of-Sums: The maxterms included are those M_j such that F() = 0 in row j of the truth table for F().



Shorthand: ? and ?

- $f_1(a,b,c) = ? m(1,2,4,6)$, where ? indicates that this is a sum-of-products form, and m(1,2,4,6)indicates that the minterms to be included are m_1, m_2, m_4 , and m_6 .
- $f_1(a,b,c) = ? M(0,3,5,7)$, where ? indicates that this is a product-of-sums form, and M(0,3,5,7)indicates that the maxterms to be included are M_0 , M_3 , M_5 , and M_7 .

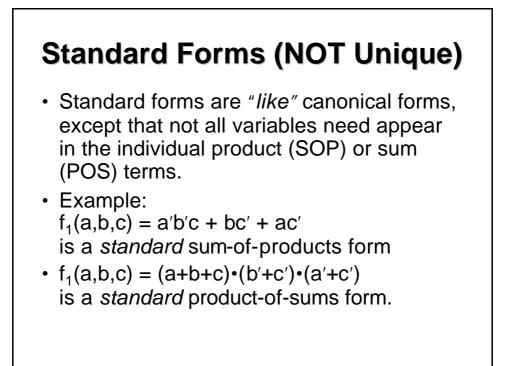
Conversion Between Canonical Forms

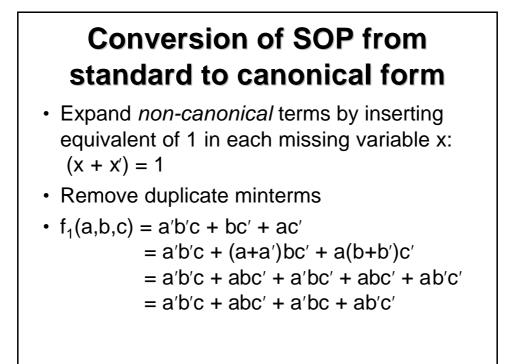
• Replace ? with ? (or *vice versa*) and replace those *j*'s that appeared in the original form with those that do not.

• Example:

$$f_1(a,b,c) = a'b'c + a'bc' + ab'c' + abc'= m_1 + m_2 + m_4 + m_6= ? (1,2,4,6)$$

= ? (0,3,5,7)= $(a+b+c) \cdot (a+b'+c') \cdot (a'+b+c') \cdot (a'+b'+c')$





Conversion of POS from standard to canonical form

- Expand noncanonical terms by adding 0 in terms of missing variables (e.g., xx' = 0) and using the distributive law
- Remove duplicate maxterms

•
$$f_1(a,b,c) = (a+b+c) \cdot (b'+c') \cdot (a'+c')$$

= $(a+b+c) \cdot (aa'+b'+c') \cdot (a'+bb'+c')$
= $(a+b+c) \cdot (a+b'+c') \cdot (a'+b'+c') \cdot (a'+b+c') \cdot (a'+b+c') \cdot (a'+b+c')$
= $(a+b+c) \cdot (a+b'+c') \cdot (a'+b'+c') \cdot (a'+b+c')$

TUGAS – 4

Sederhanakan fungsi boole berikut

a.
$$((A + B + C) D)'$$

b. $(ABC + DEF)'$
c. $(AB' + C'D + EF)'$
d. $((A + B)' + C')'$
e. $((A' + B) + CD)'$
f. $((A + B)C'D' + E + F')'$
g. $AB + A(B + C) + B(B + C)$
h. $[AB'(C + BD) + A'B']C$
i. $A'BC + AB'C' + A'B'C' + AB'C + ABC$
j. $(AB + AC)' + A'B'C$